PG – 254

I Semester M.Sc. Degree Examination, Jan./Feb. 2018
(CBCS Scheme)
MATHEMATICS
M105T : Discrete Mathematics

Time : 3 Hours Max. Marks : 70

Instructions : *i) Answer* ***any five full*** *questions.*
ii) ***All*** *questions carry* ***equal*** *marks.*

1. a) Explain methods of proof with examples.

   b) Test the validity of the following arguments.
   "If there was a cricket match, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Hence, there was no cricket match".

   c) If $S = (\sim p \vee \sim q) \rightarrow (p \leftrightarrow \sim q)$, then find its principal disjunctive normal form. **(4+5+5)**

2. a) Suppose a patient is given a prescription of 45 pills with instruction to take at least one pill a day for 30 days. Prove that there must exist a period of consecutive days which the patient takes a total of 14 pills.

   b) How many ways are there to distribute three different teddy bears and nine identical lollipops to four children ?
      i) Without restriction.
      ii) With no child getting two or more teddy bears.

   c) How many arrangements are there of TINKERER with two but not three consecutive vowels ? **(5+5+4)**

3. a) Solve the recurrence relation of the tower of Hanoi problem.

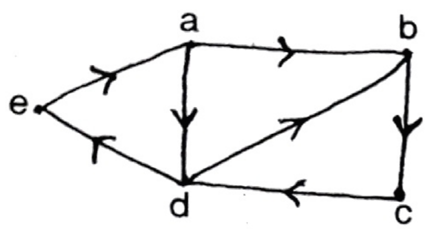
   b) Solve the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} + 9a_{n-3} = 3n^2 + 2$ with $a_0 = 2, a_1 = -2, a_2 = 4$.

   c) Using generating functions solve $a_{n+1} - a_n = 3^n, a_0 = 1$. **(4+5+5)**

P.T.O.

4. a) Define connectivity and reachability relations. Prove that $R^{\infty} = R \cup R^2 \cup \ldots \cup R^n$, for a relation R defined on a set A such that $|A| = n$.

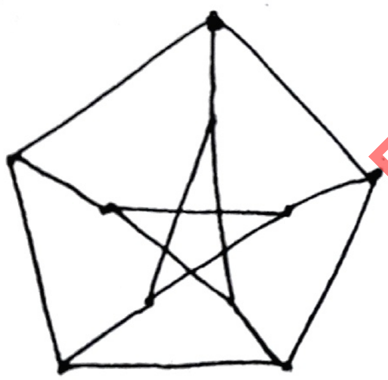
b) Define transitive closure of a relation and find the transitive closure of the relation :



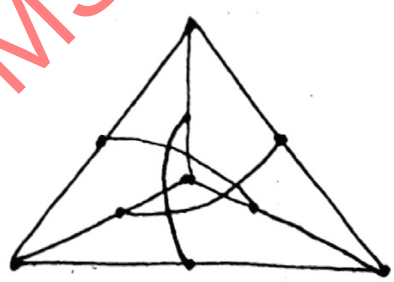
c) Define POSET. If $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be ordered by divisibility then draw the Hasse diagram of the POSET. (4+5+5)

5. a) State and prove first theorem in graph theory, further, show that in any graph G, the number of vertices in odd degree is even.

b) Define isomorphism graphs. Verify the following graphs are isometric or not.



G_1



G_2

c) Define self complementary graphs. Prove that any self-complementary graph has 4n or 4n + 1 vertices for $n \geq 1$. (4+5+5)

6. a) Let G be a connected graph. Then show that G contains an Eulerian trial if and only if G has exactly 2 odd vertices.

b) State and prove Dirac theorem for Hamiltonian graph.

c) Write a short note on the following :
   i) Konigs Berg bridge problem
   ii) Travelling salesman problem
   iii) Nearest neighbour method.

(4+5+5)

7. a) Show that any connected plane (p, q) - graph ($p \geq 3$) with r faces $q \geq \frac{3r}{2}$ and $q \leq 3p - 6$.

b) Define vertex and edge connectivity of a graph. Prove the following identity with usual notations $K(G) \leq \lambda(G) \leq \delta(G)$.

c) Define the following :
   i) Covering number of a graph $\alpha_0(G)$.
   ii) Independence number of a graph $\beta_0(G)$. Find $\alpha_0(K_p)$, $\beta_0(K_p)$, $\alpha_0(C_p)$ and $\beta_0(C_p)$.

(5+5+4)

8. a) Show that every non-trivial (p, q) – tree T contains atleast two vertices of degree 1.

b) Define binary tree with an example. Prove the following binary tree with $p \geq 3$ vertices.
   i) The number of vertices is always odd.
   ii) The number of pendent vertices is $\frac{p+1}{2}$.

c) Define minimal spanning tree. Explain Krushkal's algorithm with an example.

(3+6+5)

560

60855

I Semester M.Sc. Degree Examination, February 2019
(CBCS Scheme)
MATHEMATICS
Paper : M105T :Discrete Mathematics

Time : 3 Hours

Max. Marks : 70

Instructions : *i) Answer* ***any five full*** *questions.*

ii) ***All*** *questions carry* ***equal*** *marks.*

1. a) Explain methods of proof with examples. Prove or disprove the following statement "There is no greatest prime integer" using any of the proof techniques.

   b) Test the validity of the statements : "Either Vikram will run or Anchal will speak.
   If Anchal speaks, then Harish will fly and the rose is purple.
   The rose is not purple
   Thus, Vikram will run".

   c) Obtain the principal conjunctive normal form of the proposition $(\sim p \to r) \wedge (q \leftrightarrow p)$ without writing the truth table. Further write its negation. **(4+5+5)**

2. a) Show that there are at least 90 ways to choose six numbers from 1 to 5 so that the choices have the same sum.

   b) A big bag contains many red marbles, many white marbles and many blue marbles. What is the least number of marbles one should take out to be sure of getting at least six marbles of the same color ?

   c) How many different positive integers can be obtained as a sum of two or more numbers 1, 3, 5, 10, 20, 50, 82 ? **(5+5+4)**

3. a) Model the rabit population problem using recurrence relations and solve it explicitly.

   b) How many ways are there to distribute 25 identical balls into seven distinct boxes if the first box can have no more than 10 balls but any amount can go in each of the other six boxes ?

   c) If a leading digit 0-is permitted, find the number of 25-digit binary sequences that can be formed using an even number of zeros and odd number of ones. **(4+5+5)**

4. a) Given a set A, with $|A| = n$, and R be a relation on A. Let M be the matrix of R then prove that

   i) R is reflexive if and only if $I_n \leq M$, where I_n is the identity matrix of order n.

   ii) R is transitive if and only if $M^T \leq M$, where M^T is the transpose of M.

   iii) R is anti-symmetric if and only if $M \cap M^T \leq I_n$.

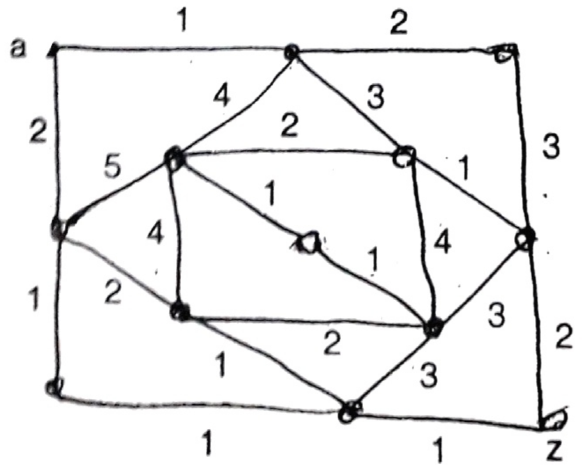
   b) Let R be a relation on a set A = {a, b, c, ,d ,e}, defined as R = {(a, a), (b, c), (a, b), (a, d), (a, e), (c, b), (b, d), (d, e), (e, d)}. Find the transitive closure of R using Warshall's algorithm.

   c) Define a boolean algebra. Show that in a boolean algebra for any two elements x and y, x = y if and only if $(x \wedge \bar{y}) \vee (\bar{x} \wedge y) = 0$. **(4+5+5)**

5. a) Define complement of a graph. Prove that two graphs are isomorphic if and only if their complements are isomorphic.

   b) Define a component of a graph. Prove that a simple graph with p vertices and K components has size atmost $\frac{(p-k)(p-k+1)}{2}$.

   c) Applying Dijkstra's algorithm, find a shortest-path between 'a' and 'z' for the following weighted graph **(4+5+5)**



6. a) Define an Eulerian graph with an example. Prove that a graph is Eulerian if and only if it can be decomposed into edge disjoint cycles.

   b) What do you mean by Hamiltonicity in graphs ? Show that any k-regular simple graph with 2k-1 vertices is Hamiltonian.

   c) Explain the nearest neighbor method using an example and hence find the weight of a spanning cycle for the considered example using the method.

(4+5+5)

7. a) Define a planar graph. Show that K_5 and $K_{3,3}$ are non-planar.

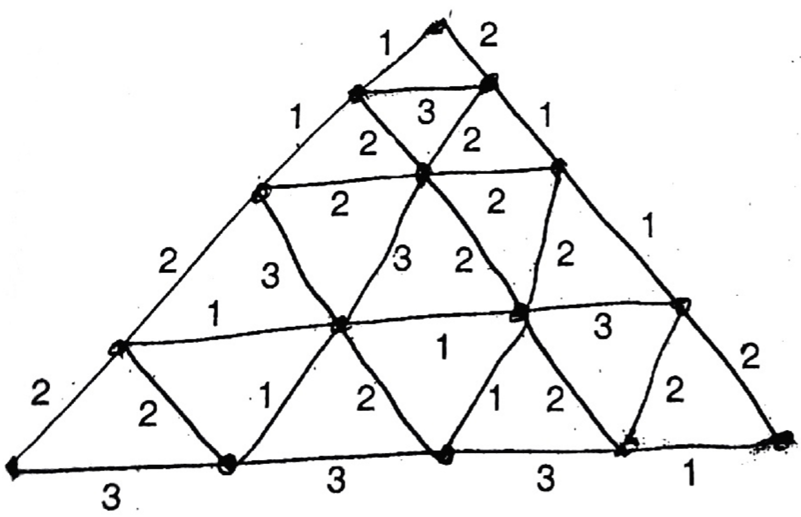
   b) If G is a triangle-free planar graph then prove that its size is atmost 2p-4, where p is the order of the graph G.

   c) Define each term of the following and establish $\alpha_1(G) + \beta_1(G) = p$. **(5+4+5)**

8. a) Define a tree. Prove that a graph with p vertices is a tree if and only if it has (p – 1) edges.

   b) Define a m-array tree. Prove that a full m-array tree with i branch vertices has mi + 1 vertices.

   c) By applying Prim's algorithm obtain a minimum spanning tree for the following graph :



(4+5+5)
